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Uncertainty Analysis in Geological Surface Modelling

Alberta Energy Regulator
AGS ALBERTA GEOLOGICAL SURVEY

Introduction

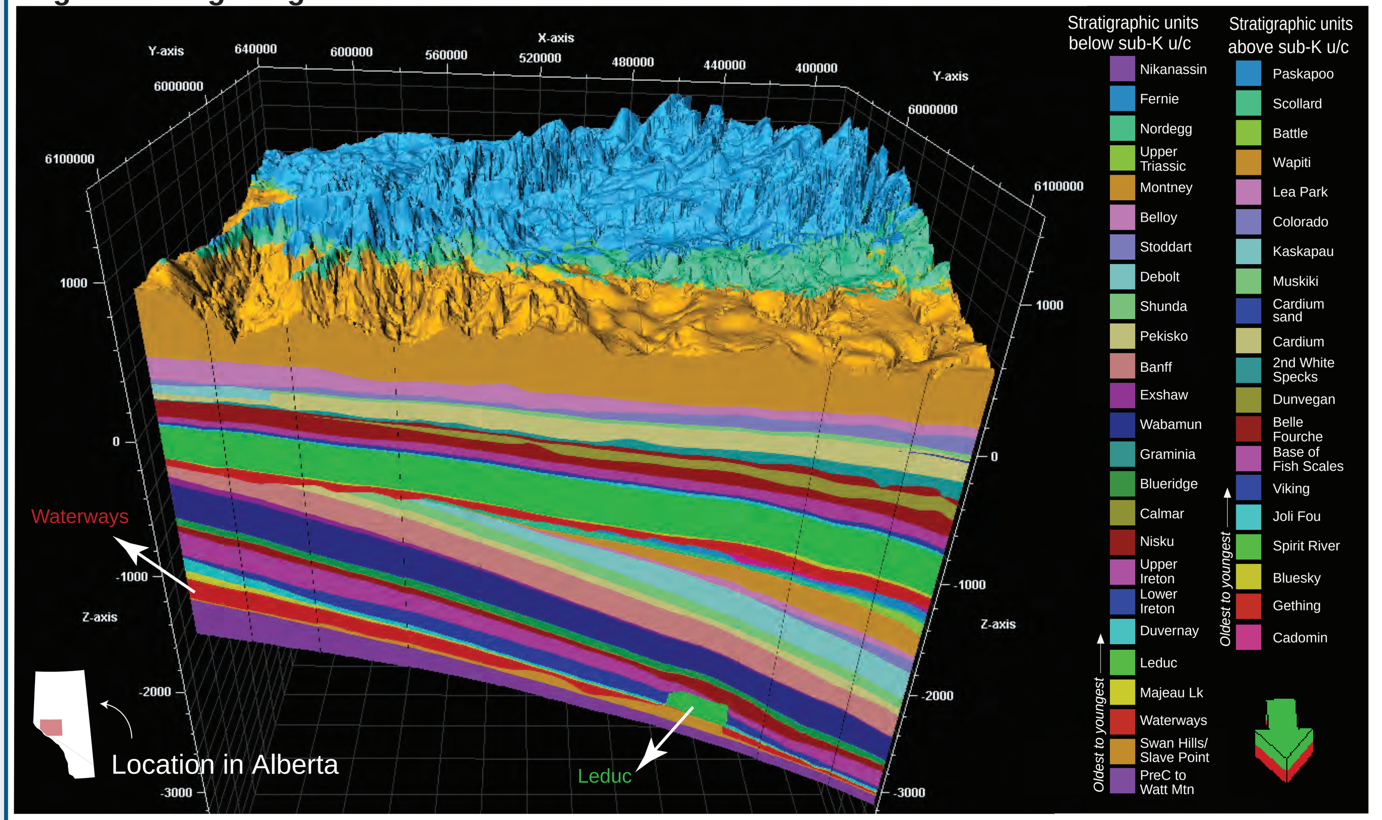
Uncertainty analysis of geological surfaces provides information about the reliability of a 3D geological model. The uncertainty in surface modelling is a result of the error in estimation which is defined as the difference between the predicted and observed values. Predictions of uncertainty can be measured by geostatistical tools defining the accuracy of the surface model to honour the available data points. A 3D geological model of the west-central Alberta (WCAB) area, was created in Petrel which includes 50 surfaces from the top of the Precambrian basement to the top of the bedrock (Figure 1). The geological surfaces were modelled using both ArcMap and Petrel; both programs have distinct benefits and shortcomings.

Surfaces modelled in ArcMap were interpolated using ordinary kriging to estimate the surface elevation at unknown locations within the study area based on the observed data points; however, these surfaces produced unrealistic results in areas of geological complexity. The convergent interpolation algorithm in Petrel was able to more realistically represent the geological complexity of the stratigraphic units (e.g., Leduc reefs; Figure 1).

Uncertainty analysis for the surfaces modelled in ArcMap can be easily obtained from the estimation standard error map that represents the standard deviation of the kriging estimate across the study area; unfortunately, it was more difficult to assess the uncertainty for the convergent interpolation results within Petrel.

To solve this problem, a unique methodology to assess the uncertainty associated with the Petrel surfaces was developed using a combination of ArcMap, Matlab, and Petrel. The cross validation of the standard deviation of multiple subset realizations of reference data is used to produce the uncertainty map of the convergent interpolation surfaces in Petrel. The local uncertainty map derived from this method shows areas of high and low uncertainty estimation on the modelled surface.

Figure 1: 3D geological model of WCAB area.



Sources of Uncertainty

1 - Data Quality
Extremely high and low values (outliers) are evaluated by geostatistical analysis. Not all the outliers are low-quality data because in some cases these outliers represent the geological features of the surface. Managing the outliers may improve the surface estimation and decrease the uncertainty in the model caused by these outliers.

2 - Data Density
The stratigraphic picks for some formations do not cover the whole WCAB area; so a major source of uncertainty in surface modelling of these formations is lack of data. Uncertainty can result from the extrapolation of surfaces in areas with sparse data.

3 - Geostatistical Model Parameters
Poor choice of geostatistical parameters and tools could cause high uncertainty in surface modelling. The proper choice of trend modelling parameters provides a variogram model that more accurately quantifies the spatial structure of the data in a statistical way. Selection of appropriate kriging parameters results in a more accurate surface modelled.

4 - Geological Complexity
Geological complexity of the surfaces is a significant, and often unavoidable, cause of high uncertainty in surface modelling. Outlier values remaining in the data set (after a quality check by geologists) are likely complex geological features of the surface such as reefs or faults.

Global and Local Uncertainty

Global uncertainty:

- Summarizes the estimation error with a number and does not give enough information about other locations (Figure 2a).

Error in estimation:

- $z_1, z_2, ..., z_n$ variables in n locations
- $\hat{z}_1, \hat{z}_2, ..., \hat{z}_n$ corresponding predicted values

$$e_i = \hat{z}_i - z_i \quad i = 1,2,...,n$$

RMSE:

- Squaring, averaging and taking square root of the estimation errors results in the root-mean-square error (RMSE)

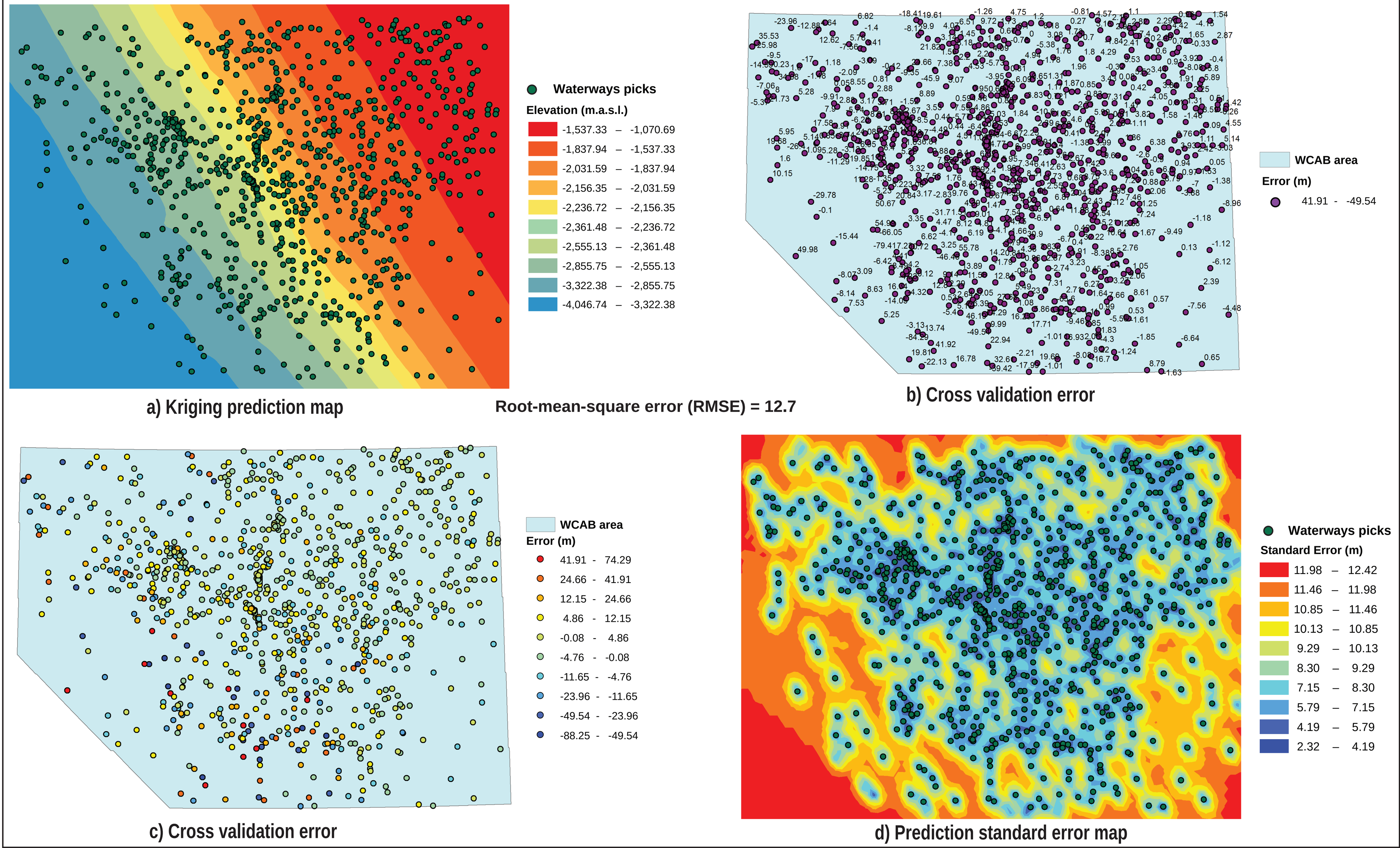
$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{z}_i - z_i)^2}$$

- Measure of the spread of estimation errors
- General parameter showing the global uncertainty of all estimation errors on the surface (Figures 2b and 2c).

Local uncertainty (Figure 2d):

- Identifies areas of low and high uncertainty of the modelled surfaces with an uncertainty map.
- Provides more information about data locations with higher error values.
- Shows estimation errors for locations with no data.

Figure 2: Different tools to show the uncertainty of the surface modelling for the Waterways top picks data.



a) Kriging prediction map — Root-mean-square error (RMSE) = 12.7

b) Cross validation error

c) Cross validation error

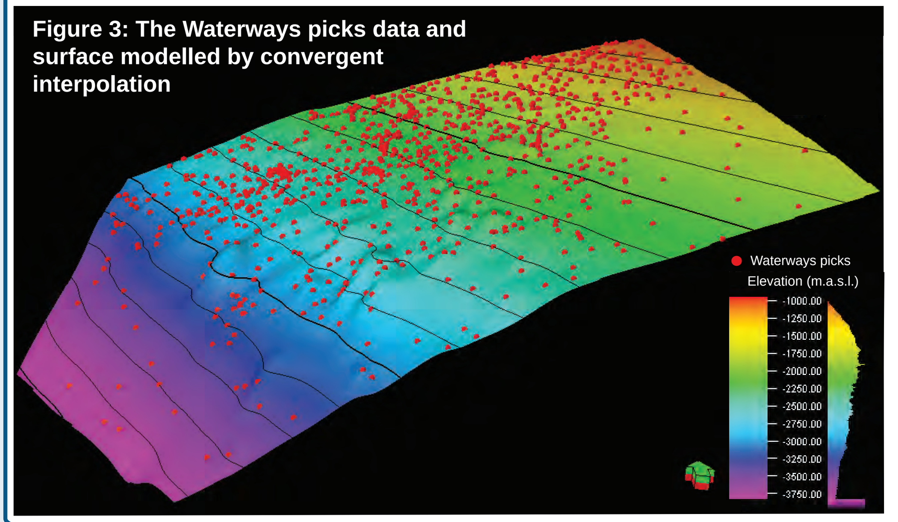
d) Prediction standard error map

Implementation of Global Uncertainty

Workflow for assessing global uncertainty of Petrel surfaces (Figure 3):

- Global uncertainty is a result of errors in estimation.
- The grid surface is converted to points to calculate the difference between observed and predicted values (Figure 4).
- Data points for each formation are not regularly spaced so the best approximation of the prediction for each data is to calculate the average of the nearest 4 estimated surface values around the observed data point.
- Global uncertainty parameters were lower for the Waterways (Figure 1) surface modelled by convergent interpolation than by ordinary kriging (Table 1).
- Estimation errors for the Waterways surface modelled by Convergent Interpolation are lower than those produce using the ordinary kriging algorithm (Figures 5 and 6).

Figure 3: The Waterways picks data and surface modelled by convergent interpolation



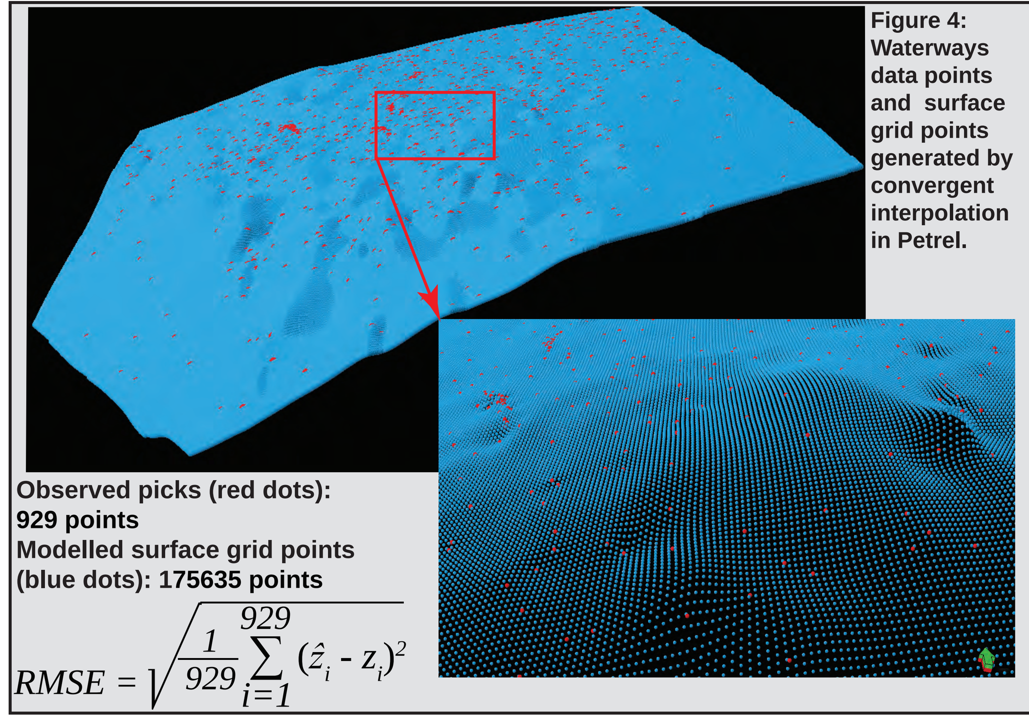


Figure 4: Waterways data points and surface grid points generated by convergent interpolation in Petrel.

Observed picks (red dots): 929 points
Modelled surface grid points (blue dots): 175635 points

$$RMSE = \sqrt{\frac{1}{929}\sum_{i=1}^{929}(\hat{z}_i - z_i)^2}$$

Table 1: Comparison of prediction errors for the Waterways Formation surface estimation.

Software	Estimation method	Mean error (ME)	RMSE
ArcMap	Ordinary kriging	-0.20	12.70
Petrel	Convergent interpolation	0.03	1.83

Figure 5: Error map of the Waterways surface modelled by ordinary kriging and convergent interpolation.

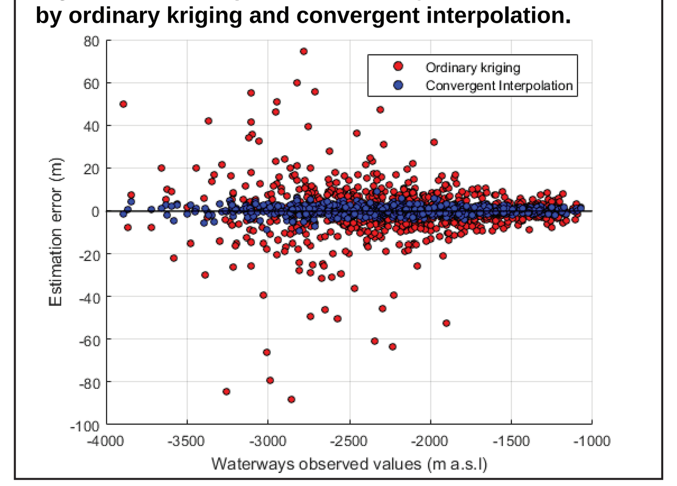
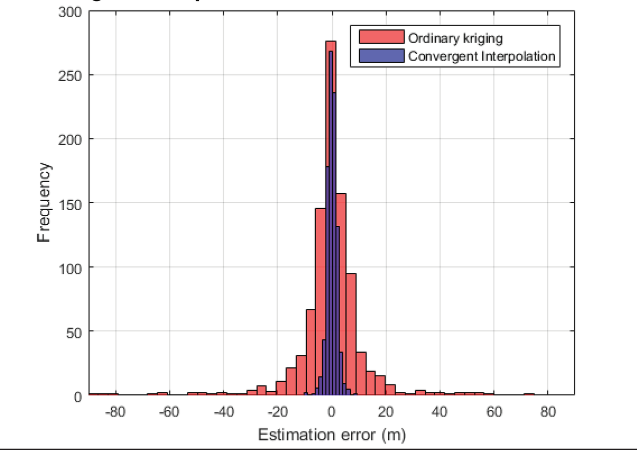


Figure 6: Histogram of estimation errors for the Waterways surface modelled by ordinary kriging and convergent interpolation.



Implementation of Local Uncertainty

The workflow for assessing the local uncertainty consists of 4 steps:

1. Generate multiple subset realizations of the reference data;
2. Import the subset realizations into Petrel to build surfaces using convergent interpolation;
3. Convert the surfaces to points;
4. Calculate the standard deviation of the surface values at each data point location for all subset realizations using a Matlab script (Figures 7 and 8).

Cross validation results were used to visualize the prediction uncertainty as a map in order to quantify the local uncertainty of the modelled surfaces generated by convergent interpolation (Figure 9). The cross validation of the standard deviation for multiple subset realizations of reference data were used to produce the uncertainty map.

Figure 7: Error map for 3 subsets of the Waterways Formation surfaces modelled by convergent interpolation.

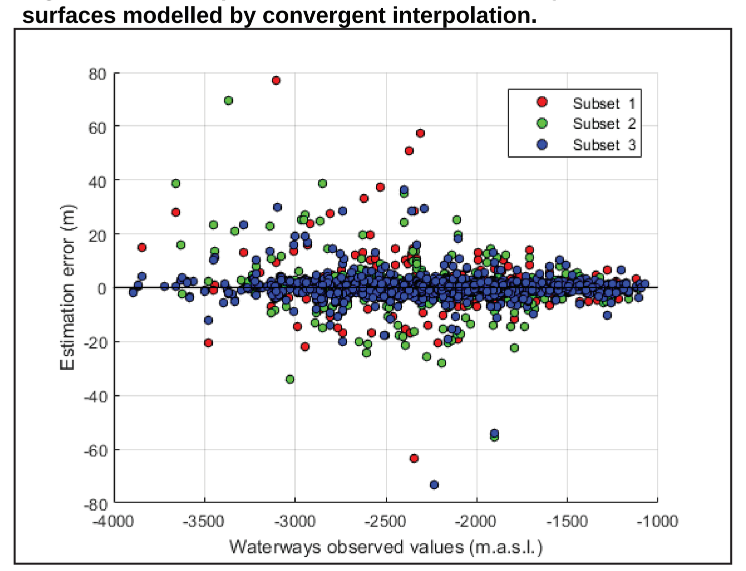
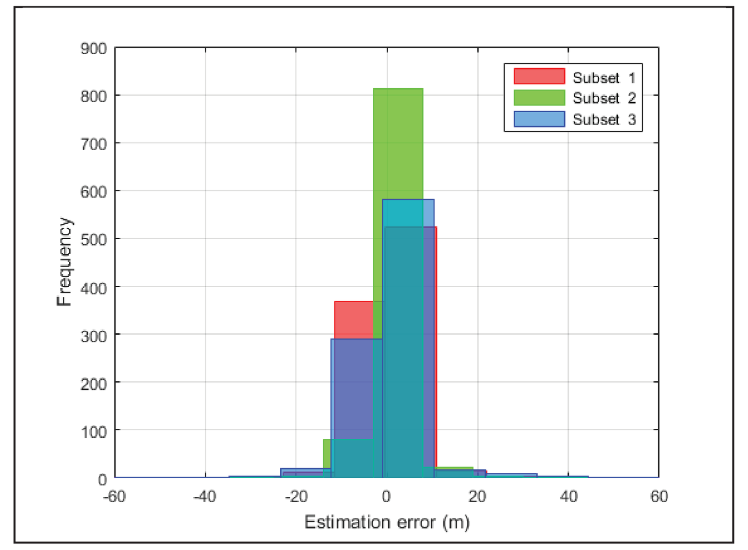


Figure 8: Histogram of estimation errors for 3 subset realizations of the Waterways picks data.

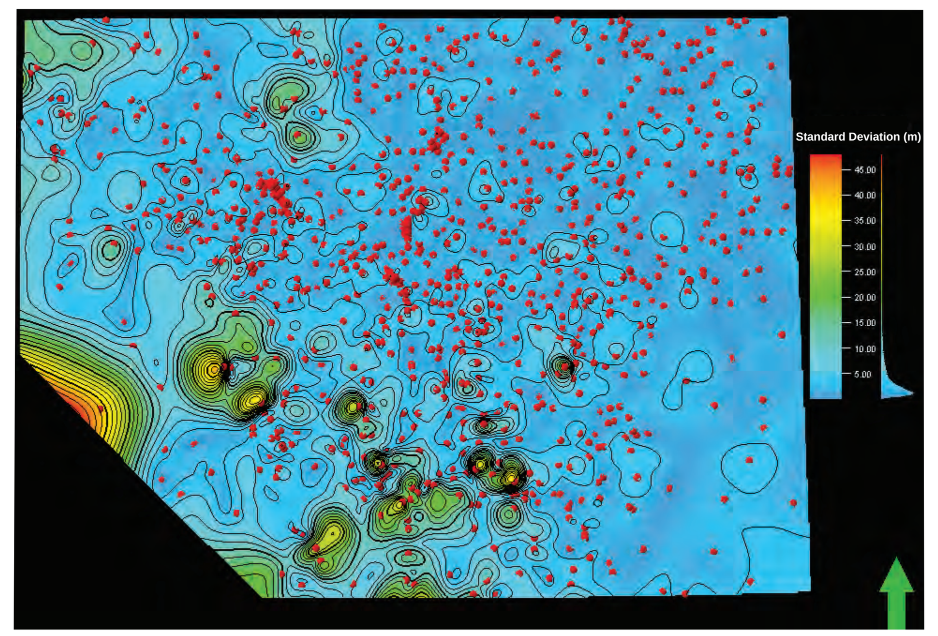


The standard deviation at each location for each of the subset realizations:

- Represents the spread of predicted values at each location.
- Can be calculated by equations 1 and 2.

Example:
Consider having 10 subset realizations for the Waterways picks data set. Each subset surface has 175,635 surface grid points. The standard deviation at each location for these 10 subsets would be calculated by Equation 3.

Figure 9: Uncertainty map based on the standard deviation for multiple subset realizations of the Waterways picks data set.



$$\sigma_i = \sqrt{E\{Z_i^2\} - (E\{Z_i\})^2} \quad i = 1,2,...,l \qquad \textbf{Equation 1}$$

$$\sigma_i = \sqrt{\left(\frac{1}{n}\sum_{j=1}^{n} z_{ij}^2\right) - \left(\frac{1}{n}\sum_{j=1}^{n} z_{ij}\right)^2} \quad i=1,2,...,l \qquad \textbf{Equation 2}$$

$$\sigma_i = \sqrt{\left(\frac{1}{10}\sum_{j=1}^{10} z_{ij}^2\right) - \left(\frac{1}{10}\sum_{j=1}^{10} z_{ij}\right)^2} \quad i=1,2,...,175635 \qquad \textbf{Equation 3}$$

Conclusion

Global uncertainty was commonly lower in the surfaces modelled by convergent interpolation in Petrel than in the surface modelled by ordinary kriging in ArcMap (Figures 10 and 11).

Prediction of the uncertainty was evaluated after modelling the surfaces in both ArcMap (Figure 12) and Petrel (Figure 13) in order to:

- Identify the areas with high and low uncertainty (Figures 14 and 15).
- Evaluate the reliability of the final 3D geological model.

The most accurate results were produced for the surface modelled by convergent interpolation in Petrel:

- Geostatistically and geologically more accurate surface modelled (Figure 13).
- Unfortunately Petrel lacks a proper tool to assess the prediction uncertainty of the modelled surfaces.
- Generating an uncertainty map of standard deviation for multiple subset realizations solved this problem (Figure 15).

Figure 10: Mean Error comparison of ordinary kriging and convergent interpolation estimation methods.

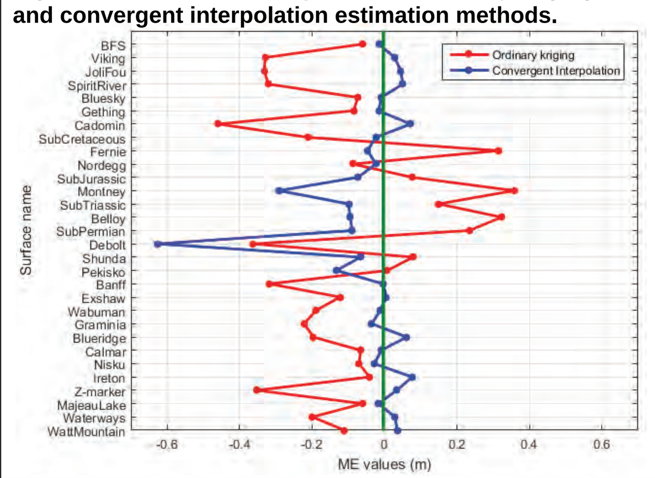


Figure 11: RMSE comparison of ordinary kriging and convergent interpolation estimation methods.

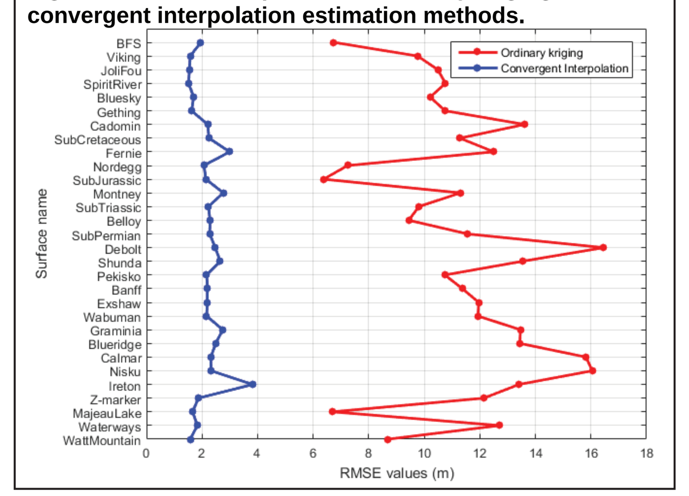
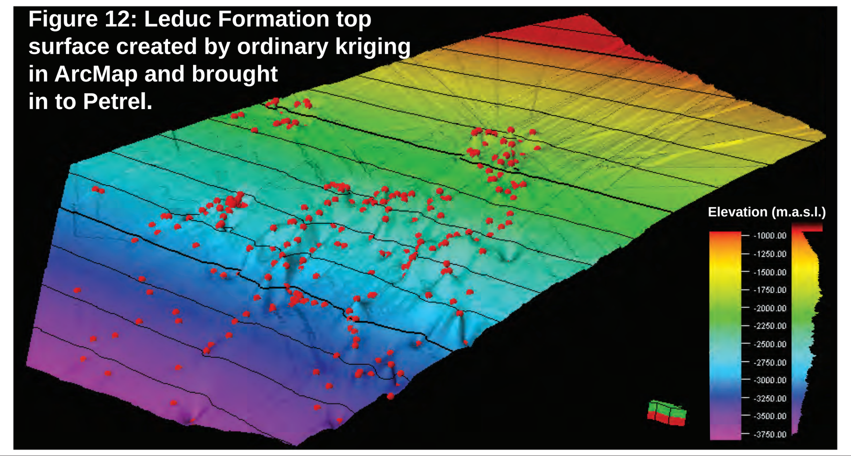


Figure 12: Leduc Formation top surface created by ordinary kriging in ArcMap and brought in to Petrel.



ArcMap

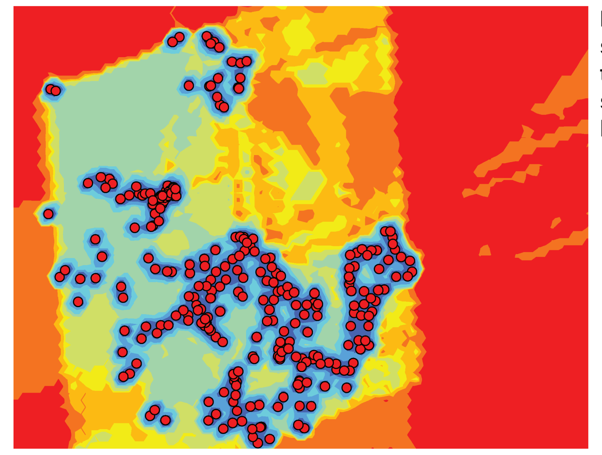
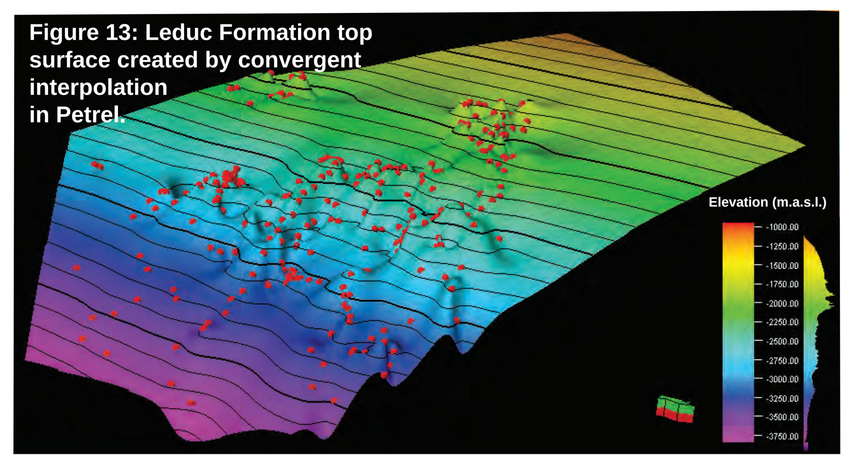


Figure 14: Prediction standard error map of the Leduc Formation surface by ordinary kriging in ArcMap.

Figure 13: Leduc Formation top surface created by convergent interpolation in Petrel.



Petrel

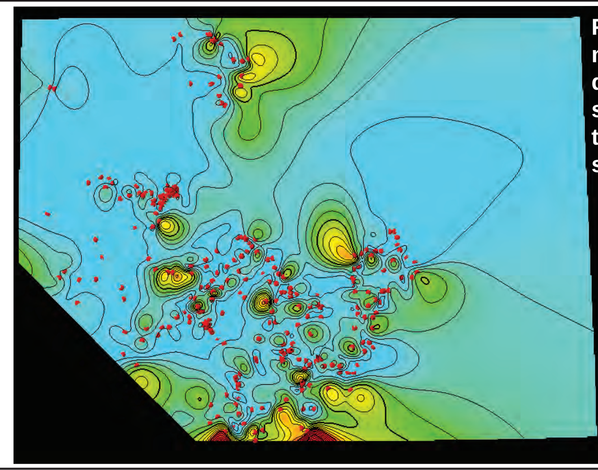


Figure 15: Uncertainty map of standard deviation of multiple subset realizations of the Leduc Formation surface in Petrel.